Rate Allocation with Lifetime Maximization and Fairness for Data Aggregation in Sensor Networks

Shouwen Lai, Binoy Ravindran
Electrical and Computer Engineering, Virginia Tech
Blacksburg, Virginia 24061, USA
{swlai,binary}@vt.edu

Hyeonjoong Cho
ETRI, RFID/USN Research Group
161 Gajeong-dong, Yuseong-gu, Daejeon 305-350, Korea
raycho@etri.re.kr

Abstract—We consider the rate allocation problem for data aggregation in wireless sensor networks with two objectives: 1) maximizing the lifetime of a local aggregation cluster and, 2) achieving fairness among all data sources. The two objectives are generally correlated with each other and usually they cannot be maximized simultaneously. We adopt a lexicographic method to solve this multi-objective programming problem. First, we recursively induce the maximum lifetime for the local aggregation cluster. Under the given maximum lifetime, we then formulate the problem of maximizing fairness as a convex optimization problem, and derive the optimal rate allocation strategy. We also present low-complexity algorithms that a local aggregation cluster can use to determine the optimal rate allocation. Our simulation results validate our analytical results and illustrate the effectiveness of the approach.

I. INTRODUCTION

We consider a wireless sensor network that is deployed in a strategic location for surveillance — e.g., tracking intruding targets [5]. Some nodes in the network generate physical measurements of an intruding target after sensing the presence of a target. A common operation on such tracking applications is data aggregation [9]. During aggregation, sensed data is gathered from different sensor nodes (i.e., source nodes) and is combined at a local cluster head. Source nodes may transmit sensed data via either one hop or multi-hop towards the cluster head.

Generally, sensor nodes are battery-powered and consume energy in sensing, transmitting, and receiving data. The limited size of sensor nodes only allows for very limited energy storage and in most applications such as tracking, it is infeasible to recharge the node batteries. Although substantial improvements have been achieved in chip design for energy conservation, energy-efficient battery designs still lag behind. Thus, one of the fundamental challenges in sensor networks is their energy efficient operation, and significant research efforts are focusing on this problem.

An important approach for achieving energy efficiency is to control the data rates in the network- or upper layers. There are several reasons for doing so. For example, by controlling the data rates, network congestion can be controlled or even eliminated [4]. Congestion, if not controlled can impede the performance of applications with delay constraints, besides wasting transmission energy and radio resources due to re-transmissions.

Further, by controlling the data rates, the network’s energy consumption can be balanced, and thereby the network lifetime can be maximized [16], [10]. For example, nodes with high remaining energy can be allowed to transmit more data, while those with low energy can be allowed to transmit less. Without such balanced energy consumption, some nodes may quickly exhaust their power, causing network partitions or malfunctions.

The another requirement, from many data aggregation applications, is to achieve fairness [3], [4] among source rates. Typically, applications can achieve better performance when data gathered from different source nodes are identical in terms of data rate. For instance, equal amount of data from some video sensor nodes can help the cluster head build a whole-scene image or video. To achieve fairness, it is important to have data rates among all source nodes as equal as possible.

In most cases, an optimized rate allocation that simultaneously maximizes the network lifetime and fairness is difficult as the two objectives are correlated with each other. For maximizing the network lifetime, it is better to bias the rate allocation for nodes with different remaining energy/transmission cost, as this will balance the energy consumption. However, for maximizing fairness, it is better to average the data rate of all nodes as much as possible. There is an inherent trade-off [17], [11] between biased rate allocation (lifetime maximization) and even rate allocation (fairness). Thus, find a rate allocation strategy to achieve such trade-off is a challenge.

In this paper, we formulate the above challenge as a multi-objective programming problem and adopt a lexicographic method [15] to solve the problem. First, we recursively induce the maximum possible lifetime in a local aggregation tree. Under the given maximum lifetime, we then formulate the problem of maximizing fairness as a convex optimization problem, and derive the optimal rate allocation strategy. We also present a low-complexity algorithm to compute the maximum network lifetime and the optimal rate allocation for fairness. To the best of our knowledge, this is the first result on rate allocation in sensor networks that simultaneously maximizes network lifetime and fairness in data aggregation using a lexicographic method.

The rest of the paper is organized as follows: In Section II, we overview the related work in rate allocation for sensor networks. In Section III, we present the network topology,
transmission power model, and our problem formulation. In Section IV, we mathematically analyze the problem model and present our lexicographic solution. Section V describes our algorithms. We report our experimental results in Section VI, and conclude in Section VII.

II. RELATED WORK

The problem of rate control and energy management in wireless sensor networks has been extensively studied. To maximize the network lifetime, Bhardwaj et al. [1] present a network lifetime upper bound for energy-efficient collaborative data gathering with optimal role assignments. Xue et al. [16] adopt a dual decomposition method to determine the optimal network lifetime for data aggregation in which source nodes have multiple routing paths to the sink node. In [13], Sankar et al. present a distributed algorithm with guaranteed approximation error for flow routing. In [7], Hou et al. study the max-min rate allocation among all nodes with a system lifetime requirement.

The problem of achieving fairness in rate allocation has also been well studied. For instance, achieving MAC-layer fairness among one-hop flows within a neighborhood is studied in [8]. In [14], the fair data collection problem with the NUM framework is studied. In [2], Chen et al. determine the maximum rate at which individual sensors can produce data without causing congestion in the network and unfairness among peer nodes.

Some previous efforts also consider both network lifetime maximization and fair rate allocation. In [11], Nama et al. present a general cross-layer framework that takes into account radio resource allocation, routing, and rate allocation for achieving trade-off between lifetime maximization and fairness. The authors solve the tradeoff problem via a dual decomposition method. In [17], the similar problem is addressed at the transport layer. The method in this work is to construct a new optimization function by linearly adding up the two objective functions (i.e., lifetime and the objective function presenting fairness) and derive an optimal solution for maximizing the newly constructed function.

The differences between our work and the above two works are: 1) we study the tradeoff problem in a local cluster which has a tree-like network topology that is more suitable for data aggregation. 2) We adopt a lexicographic method in which we prefer network lifetime maximization to fairness comparing with no preference in the existed works. This is because network lifetime is strongly correlated to energy consumption which is one of the most performance-critical aspect of sensor networks.

III. NETWORK TOPOLOGY AND PROBLEM MODEL

A. Network Topology

The network topology for data aggregation is a tree structure (aggregation tree). We have three types of sensor nodes in the network: source nodes, relay nodes, and the cluster head (i.e., root node in the aggregation tree).

The source nodes are leaf nodes which generate sensor data. The function of a source node is simple: once triggered by an event, it starts to capture live information about the target, which is then directly sent to the local cluster head within one hop or multiple hops. Only source nodes can generate data in our system. A relay node does not generate data. Its functions include: 1) receiving data from its children nodes which can be relay nodes or source nodes, and 2) forwarding the received data to the next hop toward the cluster head (root node). The cluster head is the aggregation end point.

For this network topology for data aggregation, we make the following assumptions: (1) All sensor nodes and the cluster head are time-synchronized; (2) Any sensor nodes at most has one parent in the aggregation tree; (3) Each sensor node can measure its transmission energy per byte and the remaining battery capacity; and (4) Within each cluster, the source nodes can sense events (targets) and transmit the sensed data to the cluster head simultaneously.

We also assume that interference and hidden terminal problem at relay nodes/cluster head can be avoided by virtual carrier sensing via RTS/CTS mechanism in IEEE 802.15.4 CSMA/CA protocol which is widely adopted by the MAC protocol (i.e., S-MAC or B-MAC) for sensor networks. And in the rest of the paper, for convenience, we will use the terms leaf node and source node interchangeably, and the terms root node and cluster head interchangeably.

B. Power Dissipation Model

For a sensor node, the power consumption due to data communication (i.e., receiving and transmitting) is the dominant factor. Suppose there are \(N\) sensor nodes in a cluster. Each node is denoted as \(n_i (i \leq N)\). We denote \(g_i\) as the bit rate from node \(n_i\) to its next hop node, and \(c_i\) as the transmission power cost over the radio link. We denote,

\[ w_i = \alpha + \beta \cdot d_i^m \]

where \(\alpha\) is a distance-independent constant term, \(\beta\) is a coefficient term associated with the distance-dependent term, \(d_i\) is the distance between the sensor node \(n_i\) and its next-hop node, and \(m\) is the path-loss index, with \(2 \leq m \leq 4\). Typical values for these parameters are \(\alpha = 50 mJ/b\) and \(\beta = 0.0013 pJ/b \) (for \(m=4\)) [6]. The power dissipation at the transmitter, mostly being source nodes, can be modeled as:

\[ p_s(i) = w_i \cdot g_i. \] \hspace{1cm} (1)

The power dissipation at the receiver, mostly being relay nodes or the cluster head for receiving data, can be modeled as:

\[ p_r(i) = \rho_i \cdot g_i. \] \hspace{1cm} (2)

where the typical value for the parameter \(\rho\) is 50nJ/b [6].

For a relay node, the power dissipation consists of two parts: receiving power and transmitting power. The power dissipation for relay node can be modeled as:

\[ p_t(i) = (w_i + \rho_i) \cdot g_i. \] \hspace{1cm} (3)
C. Problem Formulation

Under the tree topology, we define \( \mathcal{N} = \{ n_i | i \in [0, K] \} \) as the set of all sensor nodes. A special node in \( \mathcal{N} \) is cluster head being \( n_0 \). The set of all source nodes is denoted as \( S_0 = \{ s_k | n_k \in \mathcal{N} \} \) in which \( s_k \) is the index in \([0, K]\). In addition, we define the set of source nodes rooted at node \( n_i \) as \( S_i \) and \( S_i = \{ n_i \} \) if \( n_i \) is a source node.

Outgoing rate from source/relay node is defined as \( g_{sk} \) for node \( n_{sk} \). We also define an unified term \( c_{sk} \) which presents the energy requested for transceiving unit of data. Based on Equation 1, 2 and 3, for a source node, \( c_{sk} = w_{sk} \); for a relay node, \( c_{sk} = w_{sk} + \rho_{sk} \); for the cluster head \( c_0 = \rho_0 \).

The system lifetime, defined as \( T \), means the operational time of the local cluster until the first node runs out of power. And we define the initial remaining energy of a node as \( E_{n_i} \). Finally, the transmission capacity over the shared channel is \( R \) in this paper.

For each senor node \( n_i \) in \( \mathcal{N} \), the transceiving energy consumed within the network lifetime must not exceed its initial remaining energy. This means,

\[
\forall i \in [1, K], T \cdot c_i \cdot \sum_{s_k \in S_i} g_{sk} \leq E_i.
\]

where \( \sum_{s_k \in S_i} g_{sk} \) presents the data rate accumulated by all leaf nodes rooted at \( n_i \). For the cluster head, which is the root node of the aggregation tree, the energy consumption constraint is:

\[
T \cdot c_0 \cdot R \leq E_0.
\]

In addition, to get the better performance, the accumulated rates from all leaf nodes should not exceed the channel capacity in the cluster head. Thus, we have:

\[
\forall s_k, \sum_{s_k \in S_0} g_{sk} \leq R.
\]

Furthermore, all the rate flows must be nonnegative, and the union of all children set must be the set of all nodes in the cluster. That is:

\[
S_0 = S_1 \bigcup S_2 \ldots \bigcup S_K, \forall i \in [1, K], g_i > 0.
\]

The fairness among data rates from source nodes are defined as product of all source rates. When we maximize the product, it is equivalent to maximizing geometric mean so that we can achieve fairness. Thus, we formulate the rate allocation problem with the objective of maximizing both the network lifetime \( T \) and the product of source rates (fairness) as follows:

\[
P: \text{maximize} \quad T \prod_{s_k \in S_0} g_{sk}
\]

subject to:

\[
\text{constraints (4), (5), (6), (7)}
\]

We solve the problem by adopting a lexicographic method [15], which is a typical approach for solving multi-criteria programming problems. By this method, we first maximize one objective, \( T \), and obtain the solution space of rate vectors \( \bar{\varphi} \) for all source nodes. Within this solution space, we then derive a rate vector \( \bar{\varphi} \) to maximize \( \prod_{s_k \in S_0} g_{sk} \), and thereby seek to ensure fairness under the given maximum network lifetime.

There are two reasons to select \( T \) as the dominant objective. First, the network lifetime is strongly correlated to energy consumption, which is one of the most performance-critical aspect of sensor networks. Secondly, if we maximize \( \prod_{s_k \in S_0} g_{sk} \) first, the only optimal solution will be determined due to the convex feature of the objective function, which will make the lexicographic method ineffective.

IV. LEXICOGRAPHIC SOLUTION

A. Bit Capacity

For convenience in presentation, we introduce the notion of "Bit Capacity", which is defined as the largest amount of data that can be transmitted through one node before dissipating all its remaining energy.

Formally, it is defined as follows: Let \( B_i \) be the Bit Capacity of node \( n_i \), which is defined as:

\[
B_i = \begin{cases} 
\min \{ E_i / c_i, \sum_{d_k \in D_i} B_{dk} \}, & n_i \text{ is relay node} \\
E_i / c_i, & n_i \text{ is leaf node}
\end{cases}
\]

where \( D_i \) is the direct children set of node \( n_i \), and \( d_k \) is the index number in \([1, K]\).

For example, in Figure 1, at the initial state, \( B_0 = 20 \), \( B_1 = 7 \), \( B_2 = 4 \), \( B_3 = 5 \), and \( B_4 = 6 \). After the first iteration, \( B_1 = \min\{E_1 / c_1, B_{2}+B_{3}\} = 7 \) and \( B_0 = \min\{E_0 / c_0, B_1 + B_4\} = 13 \). Thus, the Bit Capacity of the cluster is 13.

B. Lifetime Maximization

Theorem 1: Suppose for the root node \( n_0 \), its Bit Capacity is \( B_0 \). Then the maximum cluster lifetime is given by:

\[
T_m = \frac{B_0}{R}
\]

Proof: The proof is by induction. Suppose an aggregation tree has \( H \) layers.

Base case When \( H = 1 \), Equation 10 is obviously true.

Inductive Hypothesis: Assume that Equation 10 holds when the aggregation tree has \( m > 1 \) layers. We must now show that Equation 10 also holds when the tree has \( m + 1 \) layers.

Inductive Step: For \( H = m + 1 \), suppose the children set of root node \( n_0 \) is \( D_0 \). Then, for each node \( d_k \in D_0 \), the subtree rooted at \( n_{dk} \) has at most \( m \) layers, and its lifetime is given by \( T = \frac{B_{dk}}{R_{dk}} \), where \( R_{dk} \) is the outgoing data rate from node \( n_{dk} \). Thus, \( \forall d_k \in D_0, T \cdot R_{dk} \leq B_{dk} \). Therefore,
we have \( T \leq \sum_{d_k \in d_0} B_{d_k} \). Also, for the root node \( n_0 \), its energy constraint is given by Equation 4, or expressed as \( T \cdot R \leq \frac{B_0}{T_m} \). Thus, we can show that \( T_m = \frac{R}{T} \cdot \min \{ \frac{B_0}{T_m} \} \). Therefore, we can define \( T_m \) as the maximum lifetime.

It is shown in Theorem 1 that the maximum lifetime only depends on the Bit Capacity of the root node and the channel capacity.

C. Fairness of Rate Allocation

After we obtain the maximized lifetime for aggregation cluster, we can formulate the problem of rate allocation with fairness as follows:

\[
P: \quad \text{maximize} \quad \prod_{s_k \in S_0} g_{s_k} \quad \text{subject to} \quad \forall i \in [1, K], \sum_{s_k \in S_i} g_{s_k} \leq \frac{B_i}{T_m} \quad (11)
\]

We can express the constraints as \( A \cdot \vec{g} \leq C \), where \( A \) is a matrix with \((K + 1) \times |S_0|\) dimensions and \( C \) is a vector with \( K + 1 \) items. This is a typical convex optimization problem with linear constraints, and it can be solved by some optimization methods, like Dual Decomposition [12].

However, by analyzing the problem’s constraint structure, we adopt a lower complexity method to solve the problem. Our approach is to reduce the number of constraints under the convex objective function. To understand how to address the optimization problem, we first consider the simple case in which the tree has only 2 layers.

**Proposition 1:** Suppose the aggregation tree has only two layers, and its \( K \) children are sorted as \( B_1 \leq B_2 \leq \ldots \leq B_K \). Under the maximized cluster lifetime \( T_m \), the optimal rate allocation for all leaf nodes is given by:

\[
g_j = \begin{cases} \frac{1}{T_m} \cdot \min \{ B_j, \frac{B_0}{T_m} \}, & j = 1 \\ \frac{1}{T_m} \cdot \min \{ B_j, \frac{B_0 - \sum_{j=1}^{K-1} g_j}{T_{m-j+1}} \}, & 1 < j \leq K \end{cases} \quad (12)
\]

By Lagrange relaxation theory, we can prove Proposition 1. We omit the proof in this paper due to space limitation.

In most cases, an aggregation tree has more than two layers. Our objective is to convert the constraints in Equation 11 equivalently to a constraint structure for a two-layer tree.

**Proposition 2:** Equation 11 can be equivalently reduced to the following problem which has the same constraint structure as that in two-layered aggregation tree:

\[
\text{maximize} \quad \prod_{s_k \in S_0} g_{s_k} \\
\text{subject to} \quad \sum_{s_k \in S_0} g_{s_k} \leq \frac{B_0}{T_m} \quad (13)
\]

where \( B'_{s_k} \) is the Bit Capacity value of node \( n_{s_k} \) after constraint reduction.

Before making a solid proof, we first give an intuitive explanation via the following example. In Figure 2, initially, \( B_2 = 4 \) and \( B_3 = 5 \). After one iteration, we reduce the layer of the original tree by 1. The leaf node (node 2 and node 3) will get a new Bit Capacity \( B'_{s_k} = \frac{1}{2} \cdot 7 = 3.5 \), \( B_3 = 3.5 \) (based on Proposition 1) and the node 4 will keep its Bit Capacity. After reduction, the new tree has two layers and we can apply Proposition 1 to get the final rates for all leaf nodes as: \( r_2 = \frac{1}{T_m} \cdot \min \{ 3.5, \frac{13}{1} \} = 3.5 \), \( r_3 = \frac{1}{T_m} \cdot \min \{ 3.5, \frac{13-3.5}{2} \} = 3.5 \), and \( r_3 = \frac{1}{T_m} \cdot \min \{ 6, \frac{13}{2} \} = \frac{1}{T_m} \).\( T_m \) is calculated as in Equation 10.

**Proof:** The proof is by induction. Suppose the aggregation tree has \( H \) layers.

**Base case:** \( H = 2 \). We can directly apply Proposition 1 without constraint reduction.

**Inductive hypothesis:** Suppose that when \( H = m \), the proposition holds.

**Inductive Step:** We need to show that when \( H = m + 1 \), the proposition holds. Suppose the root node has a children set \( D_0 \). For each node \( d_k \in D_0 \), if \( n_{d_k} \) is a relay node, suppose the set of leaf nodes rooted at \( n_{d_k} \) is \( S_{d_k} \). For the subtree rooted at \( n_{d_k} \) (with \( K' \) leaf nodes), since its layer is less than \( m \), based on the inductive hypothesis, the convex optimization problem can be reduced to the following problem:

\[
P'': \quad \text{maximize} \quad \prod_{s_k \in S_{d_k}} g_{s_k} \\
\text{subject to} \quad \sum_{s_k \in S_{d_k}} g_{s_k} \leq \frac{1}{T_m} \cdot B_{d_k} \quad (14)
\]

where \( B'_{s_k} \) is Bit Capacity value of node \( n_{s_k} \) after constraint reduction. Based on Proposition 1, \( \forall j \in [1, K'] \), the optimal value of \( g_j \) to maximize the fairness in the subtree is:

\[
g_j = \begin{cases} \frac{1}{T_m} \cdot \min \{ B'_j, \frac{B_{d_k} - \sum_{j=1}^{K-1} g_j}{T_{m-j+1}} \}, & j = 1 \\ \frac{1}{T_m} \cdot \min \{ B'_j, \frac{B_{d_k} - \sum_{j=1}^{K-1} g_j}{T_{m-j+1}} \}, & 1 < j \leq K' \end{cases} \quad (15)
\]

Let \( B'_{s_k} = g_j \cdot T_m \). Since \( \forall s_k \in S_{d_k}, g_{s_k} \leq g_j \), we have \( \forall s_k \in S_{d_k}, g_{s_k} \leq \frac{1}{T_m} \cdot B_{s_k} \).

The another constraint for the root node is: \( \sum_{s_k \in S_0} g_{s_k} \leq \frac{1}{T_m} \cdot B_0 \). Thus, the proposition holds.

Once the constraints are equivalently reduced to that in Equation 13, the final rate allocation vector is derived based on Proposition 1.

Since the constraint reduction is an iterative procedure, we name \( FB(s_k) \) as Fair Bit Bound for node \( n_{s_k} \) in each iteration. \( FB(s_k) \) presents the upper bound of transmitting/received bits during one intermediate iteration. For example in Figure 2, \( FB(2) = 3.5 \) and \( FB(4) = 3.5 \) after the first iteration.

V. ALGORITHMS

Based on the problem formulation and the lexicographical solution, we present an algorithms to compute the maximum
lifetime and the fair rate allocation. The algorithms contain both distributed part and centralized part. The intermediate roots of different subtrees will distributively calculate Bit Capacity and Fair Bit Bound for their leaf children. But the final maximum lifetime and optimal rate vector is calculated by the Cluster Head, in a centralized way.

Algorithm 1 shows the operation for all source nodes. It has the lowest computational complexity.

**Algorithm 1: Operations in Leaf Node (Source Node) \( n_i \):**

1. **Initialization:**
   2. \( E_i = \text{getInitialEnergy}(n_i) \);
   3. \( c_i = \text{getPowDisPara}(n_i) \);
   4. \( B_i = FB(i) = \frac{E_i}{c_i} \);
   5. Report \( \{B_i, \{FB(i)\}\} \) to its parent node;
   6. On receiving multicasted rate allocation vector \( \mathcal{F} = \{g_k\} \)
   7. If \( s_k = i \), set \( g_i = g_k \).

The operations for relay nodes and the root node (cluster head) are described in Algorithm 2 and Algorithm 3, respectively. Relay nodes and the root node first need to calculate the Bit Capacity for leaf children (line 4 of both algorithms). \( D_i \) and \( D_0 \) (in Line 3 of both algorithms) is the children set of \( n_i \), \( S_{d_k} \) is the source node in the subtree rooted at \( n_{d_k} \). Relay nodes also must update the Fair Bit Bound for all leaf children. This is shown from line 5 to line 10 of Algorithm 2. After obtaining the result of the computation, they report the result to their parent nodes for further iteration. A relay node also relays the multicasted rate vector from its parents to its children. The root node calculates the optimal rate vector after getting information from all the leaf nodes (source nodes), and send back the fair rate allocation via multicasting to all leaf nodes (line 13 of Algorithm 3).

**Algorithm 2: Operations in Relay Node \( n_i \):**

1. **Initialization:**
   2. \( E_i, c_i \) and \( B_i \);
   3. On Receiving Reports \( \{B_{d_k}, \{FB(s_j)\} | S_{d_k}| \} \) \( d_k \) \( D_i \);
   4. \( B_i = \min\{B_i, \sum_{j \in S_{d_k}} B_{s_j}\} \);
   5. Sort the members in \( \{FB(s_j)\} | S_{d_k}| \) \( d_k \) \( D_i \);
   6. \( \{FB(j)\} = \) The sorted set in which \( B_{j-1} \leq B_j \);
   7. \( \text{sum} = 0 \);
   8. for \( k = 1 \) to \( |S_i| \) do
     9. \( FB(k) = \min \{FB(k), B_{|S_i|+\text{sum}}\} \);
    10. \( \text{sum} = \text{sum} + FB(k) \);
   11. Report \( \{B_i, \{FB(s_j)\}\} \) to the parent node;
   12. On receiving multicasted rate vector \( \mathcal{F} \);
   13. Forward the information to all subtrees via multicast;

**Theorem 2:** Algorithms 1, 2, and 3 determine the optimal rate allocation to maximize network lifetime and to achieve fairness under the given maximized lifetime.

**Theorem 3:** Algorithms 2 and 3 have a complexity \( O(n \log n) \), and the upper bound of the delay overhead is \( K \cdot RTT \). \( K \) is the number of all nodes in the aggregation tree and \( RTT \) is the average round trip time over one hop transmission.

Due to space constraints, we don’t provide detailed proofs of the two theorems. Theorem 2 directly follows from the two algorithms that determine the optimal rate allocation based on our analysis in Sections IV-B and IV-C. Theorem 3 directly follows from the algorithm structure.

**Algorithm 3: Operations in Root Node (Cluster Head) \( n_0 \):**

1. **Initialization:**
   2. Set value for \( E_i, c_i \) and \( B_0 \);
   3. On Receiving Reports \( \{B_{d_k}, \{FB(s_j)\} | S_{d_k}| \} \) \( d_k \) \( D_0 \);
   4. \( B_0 = \min\{B_0, \sum_{j \in S_{d_k}} B_{s_j}\} \);
   5. \( T_m = B_0 \);
   6. Sort the members in \( \{FB(s_j)\} | S_{d_k}| \) \( d_k \) \( D_i \);
   7. \( \{FB(j)\} = \) The sorted set in which \( B_{j-1} \leq B_j \);
   8. \( \text{sum} = 0 \);
   9. for \( k = 1 \) to \( |S_0| \) do
     10. \( g_k = \frac{1}{T_m} \cdot \min \{FB(k), B_0 + \text{sum}\} \);
    11. \( \text{sum} = \text{sum} + FB(k) \);
   12. \( \mathcal{F} = \{g_k \} | S_0| \);
   13. Multicast \( \mathcal{F} \) to all source nodes;

VI. EXPERIMENTAL RESULTS

We evaluated the effectiveness of our algorithms through simulation-based experiments. We first randomly generated an aggregation tree with a topology as illustrated in Figure 3. The number of children of non-leaf nodes was randomly distributed between \([0,5]\).

![Fig. 3. Topology in Experiments](image)

The distance between one node to its next hop node was also randomly generated between \([15,30]\) (m). In our experiments, we set \( \alpha = 50mJ/\beta = 0.0013pJ/b/m^2 \), and \( m = 4 \) for the power consumption model. The initial energy reserve of each sensor node was defined using a normal distribution with mean and variance of \((25J, 16J^2)\). The shared channel capacity (IEEE 802.15.4) is set to 128Kb/s in our experiment.

![Fig. 4. Topology in Experiments and Solution Space for All Rate Vectors](image)

To illustrate our solution strategy for the multi-objective programming problem, we show the entire solution space in Figure 4. Each data point in the figure corresponds to one rate vector (for all source nodes). The value of network lifetime
and the product of all source data rates were calculated for each vector.

We randomly generated 500 rate vectors and plot the network lifetime $T$ and $\prod_{k\in\mathcal{S}_i} g_{s_k}$ for all vectors. Our goal is to find the vector point (marked by red color) in the most upper-right corner of Figure. 4. This most upper-right point present the rate vector with maximized network lifetime and maximized fairness under the given maximum lifetime.

We show our rate allocation strategy in Figure. 5. The rates were calculated based on the distributed algorithm in Section V. By comparing with the Average Rate Allocation strategy in which all source nodes have same data rates, we observe that the rates in our strategy varies from node to node since each node has different Bit Capacity.

Figure. 6 shows the individual lifetime for all sensor nodes. Recall that the network lifetime is defined as the smallest lifetime among all the nodes. From the figure, we observe that our rate allocation strategy achieves longer network lifetime than the average rate allocation strategy.

Figure. 7 shows the lifetime of the same cluster when we change the node configuration with different remaining energy and transmission distance. The remaining energy and transmission distance of each node in different experiments has a normal distribution. We repeated the experiment for 60 times, and drew the maximum network lifetime in each time for our rate allocation strategy and the average rate allocation strategy. From the figure, we observe that the maximum lifetime also has a normal distribution. In addition, our rate allocation strategy always achieves better performance than the average rate allocation strategy.

**VII. CONCLUSIONS**

This paper studies how to maximize the cluster lifetime and to achieve fairness with rate control for data aggregation applications on sensor networks. To solve the multi-objective programming problem, we adopt a lexicographic method by first determining the solution space of lifetime maximization and then deriving the optimal rate allocation strategy for fairness under that solution space. We also present low-complexity algorithms to compute the maximum lifetime and the optimal rate vector for fairness. The simulation results illustrate the effectiveness of the approach.

Several directions exist for further study, including rate allocation with multi-target tracking and multi-path routing for lifetime maximization and fairness.

**REFERENCES**


